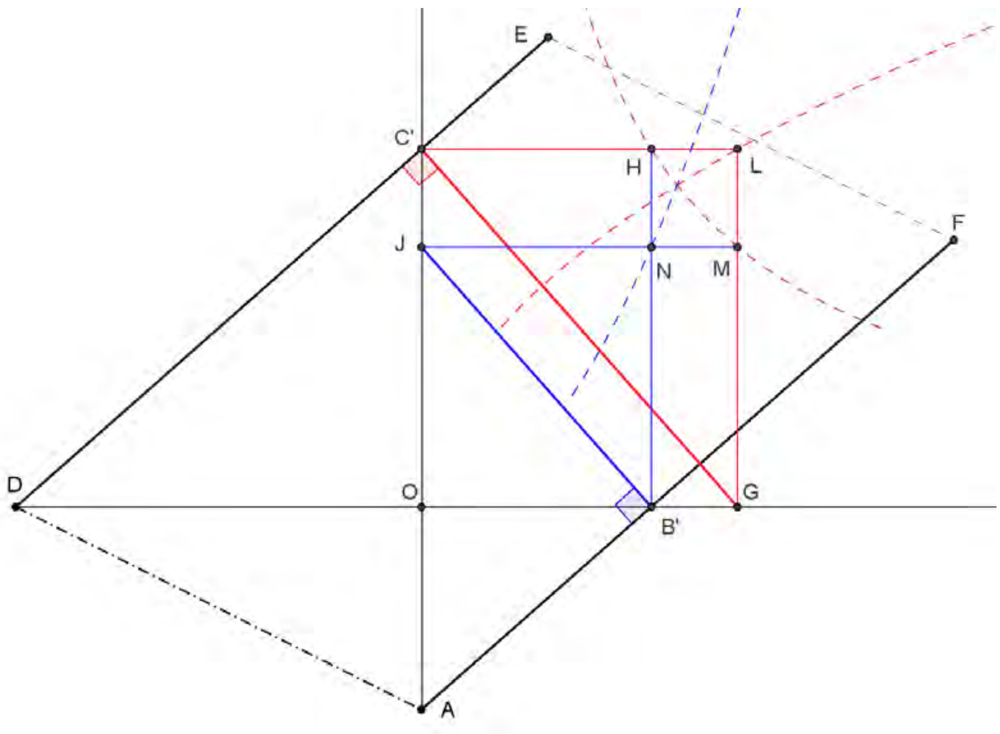
Is This How Menaechmus Solved the Duplication of the Cube Problem?

A device is discussed here that could plausibly have been constructed in Menaechmus' time for drawing two intersecting curves to solve the cube duplication problem. I have loosely defined it as a compass though it might be better described as a parallelogram version of Plato's frame.

In this figure taken from a GeoGebra model, ADEF is the parallelogram with points A and D fixed so that OD:OA = 2:1. DE and AF rotate about the points D and A respectfully and remain parallel as they rotate.



C' is constrained to move along both DE and AO extended while B' is constrained to move along AF and DO extended as the sides of the parallelogram rotates while J and G are constrained to move along AO and DO extended as B' and C' move.

B'H is perpendicular to DO extended and moves with B'. GL, C'L and JM are attached and behave similarly. Pens located at H, L, M and N trace the curves shown as the sides of the parallelogram rotate with H and M tracing the same curve.

Using the property of parallel lines it can be shown that OC'*OB'= OG*OJ = DO * AO. This leads to the observation at for any point on the curve traced by H (or M), the area in the rectangle formed by AO and DO extended and the two lines dropped perpendicular from the point to AO and DO extended is constant.

When points H, L, M, and N are congruent JB' and C'G will be collinear and OJ (or OC') and OB' (or OG) will be the two means between DO and AO.

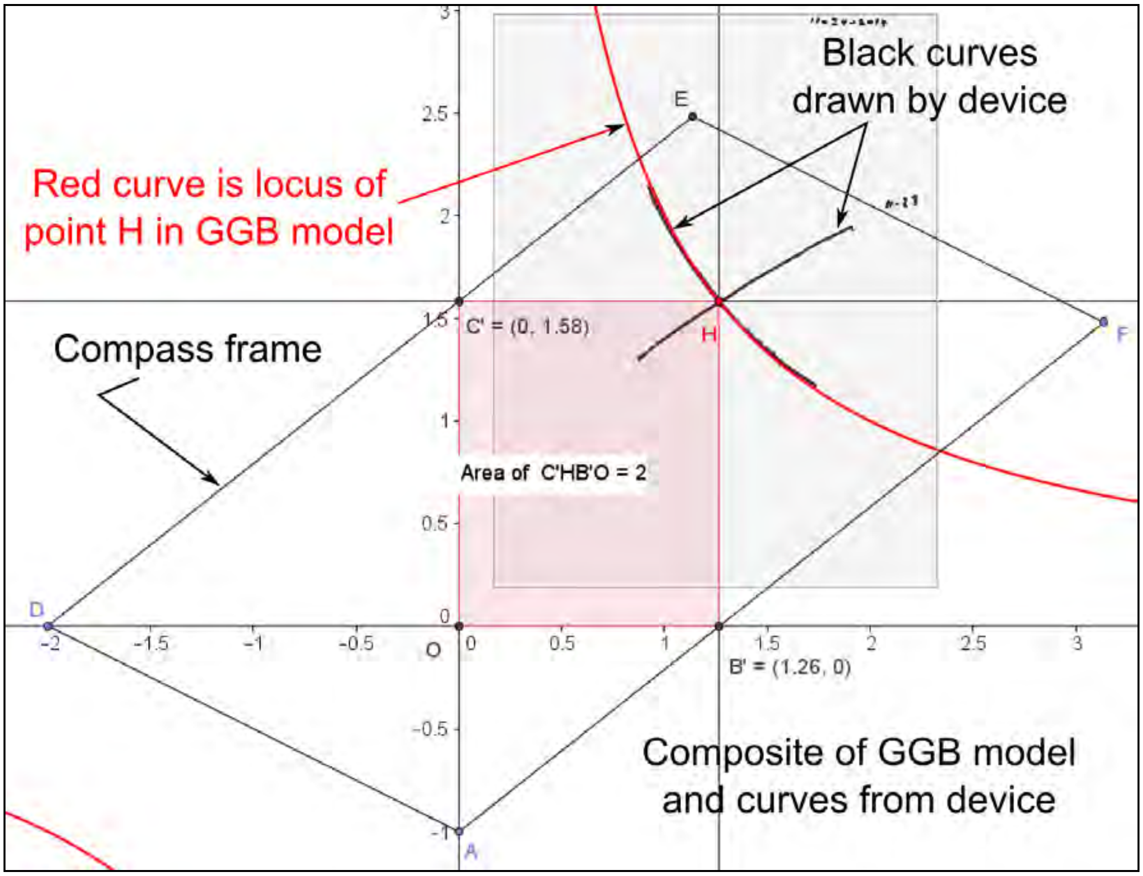
Thus the perpendicular lines from the point of intersection of the three curves to AO and DO extended are also the means.

Only two of the curves are actually required to find the means. I chose to use pen H for one curve and L for the other. The line C'G, JB', JM and GL are not needed and were not include in the device constructed and shown here.

For the second curve drawn with pen L the device was reconfigured without lines AF, JB', B'H and JM and the curve was then drawn on the same sheet of paper. That configuration is not shown here.

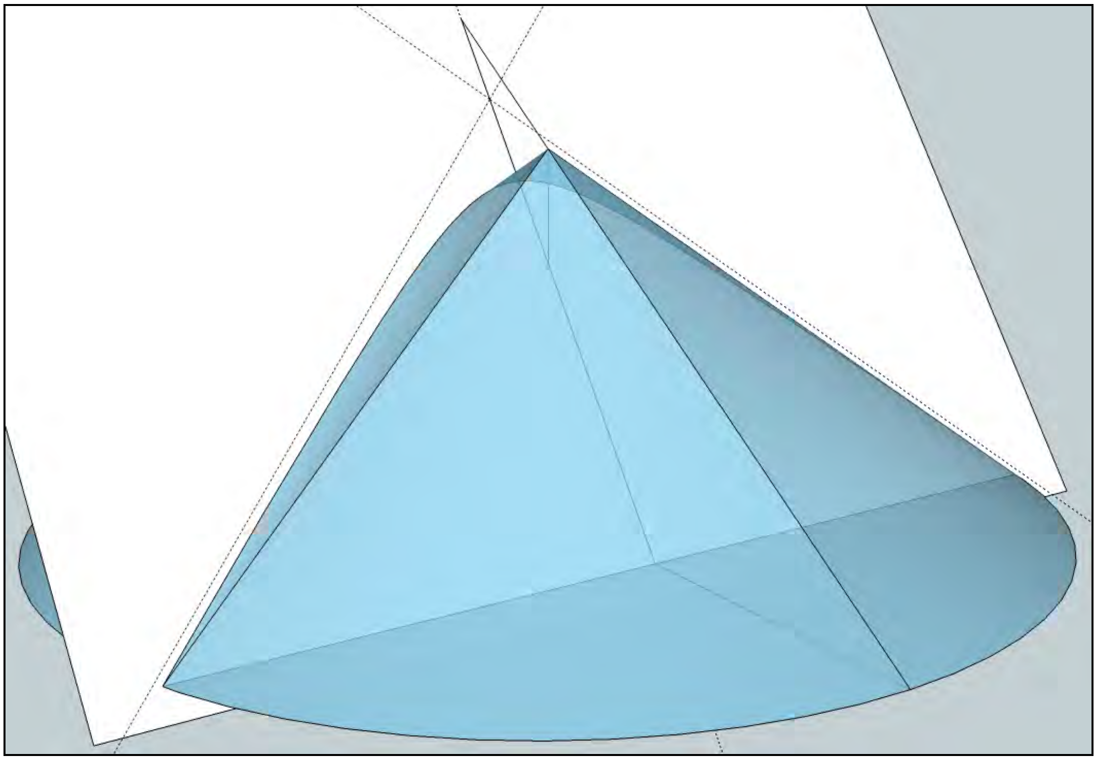
The drawing sheet from the device was scanned and inserted into the GGB model discussed above and shown in the next figure.

The red curve is the curve as drawn by point H of the GGB model. The two curves drawn by the device pens H and L are shown in black with the drawing sheet in the background.

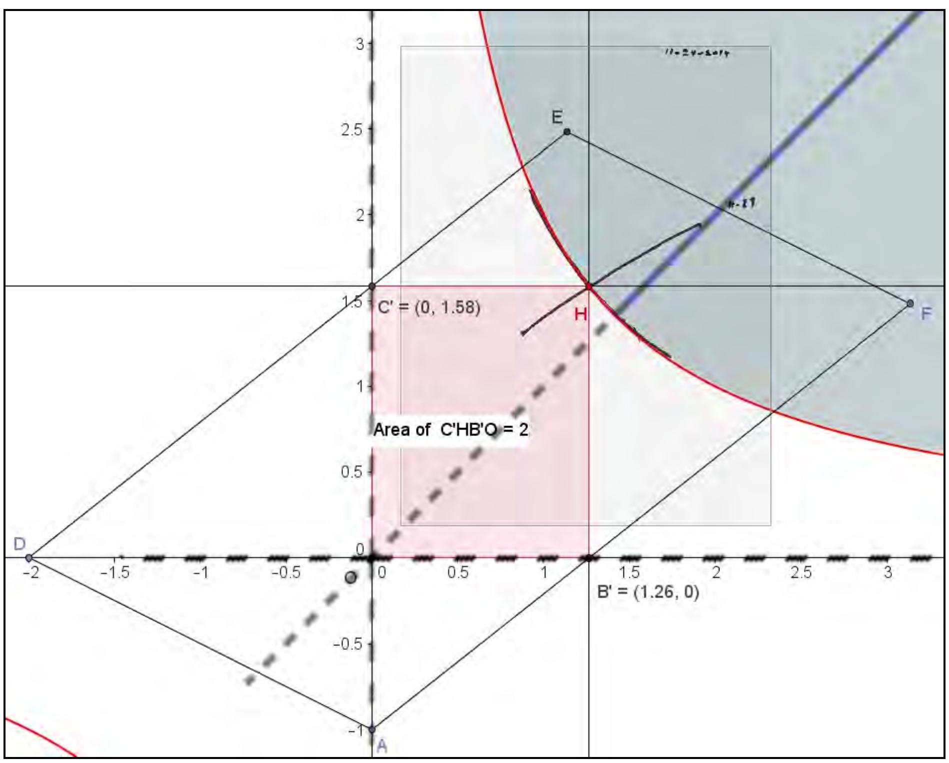


No one knows how Menaechmus solved the cube duplication problem but this demonstrates a solution that would have been feasible his time. Even if he did use such a method we are left wondering if he recognized that the curves were conics. It would seem likely that he would have recognized the curves as being the same as those that could be constructed using the properties of the sides of right triangles and studied using the method of application of areas but how or who discovered they were sections of an obtuse-angled and a right-angled cone may never be known.

It is an interesting and enlightening exercise to create the curve described as a "section of an obtuse-angled cone" using a right triangle. Then the diameter and parameter can then be found using the method of application or areas and used to determine the obtuse angle and distance from the apex of the cutting plane to produce the required section. Such a cone and the resulting section created in SketchUp are shown on the next page.



When the section is extracted and added to the GGB model it fits the other curves nicely.



It would be interesting to know if Eratosthenes' comment "Nor cut from a cone the Menaechmian triads" was an indication of how Menaechmus actually constructed his curves.
jhmc 2014-11-28